

STORAGE

THE EISENHOWER LIBRARY



3 1151 02676 8519

54.351

Library



Johns Hopkins ^{of the} University



accepted May 25/91

The Orbit of Hyperion.

A Thesis

*Submitted for the Degree of Doctor of Philosophy
at the Johns Hopkins University.*

By

W. S. Eichelberger.

1891.

Table of Contents.

	Page
§ 1 Introduction.	1
§ 2 Elements of Hyperion	4
§ 3 Elements of Alcyon	5
§ 4 Comparison of Observations with Assumed Elements	12
§ 5 Equations of Condition	20
§ 6 Normal Equations and their Solution	37
§ 7 Final Elements	48
Perigee distance	49
Eccentricity	53
Mean Longitude	54

Hyperion

§1. Introduction

On the 19th of September, 1848, an eighth satellite of Saturn was discovered by Wm. W. Bond of Cambridge, Mass. and on the same evening, by Mr. Lassell of Liverpool.

Only a few observations were made of it in the first quarter of a century after its discovery, but later observations of it were made. About fifteen years ago, however, Professor Asaph Hall, by the aid of the 26-inch equatorial at the U. S. Naval Observatory, began a series of observations, which extends to the present.

Attention was first called to the peculiarity of its motion by Professor Galle's paper in the Astronomische Nachrichten, and also in the Astronomische Zeitschrift. In the latter paper he points out the fact that the peri-saturnium of Hyperion has a nearly

retrograde motion of $20^\circ \pm$, and adds, that
"on account of its mean distance and the
eccentricity of its orbit, Hyperion can approach
very near to Titan; and since three times the
period of one is nearly equal to four times
the period of the other, there is probably a
large perturbation of the motion of Hyperion
by Titan."

Professor Newcomb shows that this remarkable
retrograde motion of the peri-saturnia is
due to the mutual attraction of these two
satellites, and obtains the perturbations of the
elements of the motion of the peri-saturnia.
The value that
he deduces for the mass of Titan, after correcting
the value obtained by the $\frac{1}{2}$ method, is

1. On the Motion of Hyperion &c. &c. in Celestial
Mechanics. A. P. American Ephemeris, 1881.
2. Astronomical Journal, no. 176. $m = \frac{1}{4714}$

M. F. Tisserand has written an article in which he considers the perturbative effect of two bodies revolving in circular orbits about their primary, and shows, that if the mean motions are in a ratio that approaches very near commensurability, and the mass of one is very small compared with the other, the orbit of the smaller will become quite eccentric, and its peri-centre will retrograde. Since Titan, from its greater brilliancy, is considered much larger than Iapetus, the satellites afford him an actual example, and assuming the eccentricity of the latter's orbit to be .01 and that it is entirely due to the perturbative effect of Titan, he deduces a value of the mass of Hyperion $m = \frac{1}{10750}$

The subject has been thoroughly treated by Tisserand.
 1. Tisserand. Sur un Cas Remarquable du Problème des Perturbations. Comptes-Rendus, 1886. 2^e Semestre, p. 446.
 2. Annals of Mathematics, Vol. III, p. 161, and Vol. IV, p. 53. $m = \frac{1}{4617}$

and Hill,[†] by different methods, both deriving nearly the same mass for Titan.

In all these cases the orbit of Titan has been so small a mass, that the perturbing effects. In considering this problem it is my intention to take account of these small quantities. As a preliminary step, it was determined to obtain more accurately the elements of Hyperion, and this problem has so grown that the present paper will be confined to its consideration.

§2 Elements of Hyperion.

To determine the elements of Hyperion, no more was had to the Washington Observations. About twenty were taken from each of six oppositions, during the years 1855-56, and a small number of the first of the series. The elements were determined by the method of least squares, and the results are given in the following table.

places, the provisional elements were obtained mostly from Professor Bencomb's paper, the plane of the orbit being supposed to coincide with that of Titan.

The observed data were reduced to the time of the observation, of the position, angle, and from these were then computed the rectangular co-ordinates of the satellite referred to the parent moon centre.

The formulae used for computing the tabular places are the following:

$$\xi = r f p_0 \Delta \sin f \sin (F + u)$$

$$\eta = r f p_0 \Delta \sin g \sin (G + u)$$

$$\zeta = r f p_0 \Delta \sin h \sin (H + u)$$

$$x = s \sin \phi = \frac{\xi}{1 + \xi}$$

$$y = s \cos \phi = \frac{\eta}{1 + \xi}$$

$f, g, h, F, G,$ and H being given by the following:

$$\sin f \sin F = \sin (\alpha - N)$$

$$\sin f \cos F = \cos (\alpha - N) \cos \beta$$

$$\begin{aligned} \cos f &= -\cos(\alpha - A) \sin \delta \\ \sin g \sin \delta &= -\sin \delta \cos(\alpha - A) \\ \sin g \cos \delta &= \cos(\delta + L) \\ \cos g &= \sin(\delta - Q) \\ \sin h \sin A &= \cos \delta \cos(\alpha - A) \\ \sin h \cos A &= \sin(\delta + L) \\ \cos h &= \sin(\delta - Q) \\ \sin L &= \cos f \sin(\alpha - A) \\ \cos L &= \sin f \\ g \sin Q &= \sin f \sin(\alpha - A) \\ \cos Q &= \cos f \end{aligned}$$

The following notation has been employed.

λ = Mean Longitude of Satellite

40 = Heart Anomaly

λ = Longitude of Node on the Equator.

i = Inclination of Orbit to Equator

- e = Eccentricity of Orbit
 μ = Mean Motion in Longitude
 Δ = Mean Distance of Satellite from Planet, as seen at
~~distance p_0 from the Planet~~
 p_0 = Assumed Mean Distance of Planet from Sun.
 p = Heliocentric Distance of Planet.
 α = Right Ascension of Planet
 δ = Declination of Planet
 v = True Anomaly of Satellite
 ε = Eccentric Anomaly
 u = Longitude of Satellite in Orbit, starting from
 Node on the Equator.
 s = Observed Distance of Satellite.
 ϕ = Observed Position Angle
 w = Distance along Orbit from Node on Ecliptic to
~~Node on Equator.~~

The values of f, g, h, T, Y and H were computed
 for every five days during the periods of observation,
 the values for each observation being found by

interpolation.

The value of $\bar{Q}$ for each opposition is the mean of the values obtained from two, or three consecutive observations of the comet in series, by means of the following formulae

$$\tan u = \frac{s \cos p \frac{\sin F}{\sin \gamma} - s \sin p \frac{\sin \gamma}{\sin f}}{s \sin p \frac{\cos \gamma}{\sin f} - s \cos p \frac{\cos F}{\sin \gamma}}$$

$$v = u + n - w - P$$

$$M = v - \text{Equa. of Centre}$$

$$\bar{Q} = Q + \dots$$

§ 3. Observations of Comet

Time	γ	γ	Mean Time	γ	γ
Aug. 1. 1172	267°. 81	193". 94	Sept. 11. 5958	110°. 21	145". 71
Aug. 1. 1121	265. 30	117. 74	19. 5003	211. 77	204. 11
Aug. 1. 5754	277. 81	121. 18	23. 5577	287. 71	173. 72
Sept. 1. 3863	87. 37	216. 00	27. 5037	74. 0	232. 16
Sept. 1. 1171	77. 11	227. 62	Oct. 2. 3196	128. 41	166. 74

Observations of Venus (1877)

1876			1877		
Oct. 3. 4777	118. 11	117. 74	Nov. 27. 4330	94. 05	200. 27
17. 4712	75. 59	142. 67	29. 4266	95. 15	137. 20
19. 4713	88. 20	209. 86	30. 4241	102. 73	13. 12
20. 4527	92. 10	223. 96	Dec. 6. 4332	273. 92	220. 97
21. 4917	96. 21	226. 71	7. 4331	275. 28	222. 71
22. 4728	100. 01	210. 94	8. 3993	276. 12	207. 48
24. 4254	112. 10	134. 73	9. 4138	277. 65	180. 16
28. 4313	201. 00	134. 33	14. 4613	87. 06	122. 26
29. 4305	268. 50	174. 73	15. 4408	88. 07	120. 29
1877			1877		
Oct. 2. 5856	273. 40	225. 98	Oct. 10. 5970	90. 70	220. 51
12. 5438	89. 02	151. 84	12. 6390	92. 02	148. 86
13. 5271	91. 31	191. 92	13. 5883	73. 80	93. 78
14. 4962	92. 99	215. 49	16. 6508	285. 48	123. 88
14. 5234	12. 98	213. 58	17. 5660	274. 67	277. 77
15. 5042	94. 27	219. 58	20. 5641	272. 52	260. 34
Nov. 6. 4758	15. 35	214. 52	23. 5840	265. 65	190. 60

Oct. 24. 5005	260.00	104.51	Oct. 16. 5793	270.70	237.31
31. 5322	71.22	250.00	23. 5145	107.35	141.69
1. 5076	27.10	102.76	24. 5637	99.80	189.76
10. 5177	273.65	222.00	25. 5799	94.20	222.38
11. 4000	271.41	222.46	26. 5332	12.59	59.76
2. 5115	267.20	234.70	Nov. 1. 5651	315.60	80.76
13. 4577	207.29	177.00	2. 5439	274.53	129.86
14. 4632	263.27	149.75	7. 5383	268.60	248.06
21. 4727	222.20	213.00	9. 5640	259.03	176.46
22. 4500	277.00	170.00	11. 5609	240.25	61.17
30. 4348	215.07	222.01	12. 5542	141.67	60.66
			16. 5414	90.90	221.21
Oct. 10. 6214	252.00	62.11	17. 4983	87.37	224.23
11. 6007	257.75	96.28	20. 5209	63.77	156.90
12. 6034	271.53	147.29	21. 5055	30.77	62.00
14. 5834	270.17	230.00			
15. 5678	274.35	253.22	22. 5033	12.50	271.71

Observations of the Moon (continued)

Time	Lat.	Long.	Time	Lat.	Long.
Dec. 2. 6172	73.00	222.57	Dec. 7. 5774	27.1.10	220.10
3. 5777	61.47	226.17	8. 5727	26.9.17	223.28
2. 6167	10.65	140.47	9. 5794	260.39	225.10
6. 5625	13.20	111.21	15. 5784	123.98	62.24
7. 5617	217.23	181.53	19. 5685	87.10	220.22
10. 5377	277.73	213.85	26. 5103	327.06	114.10
17. 5212	132.25	133.51	1885 Jan. 2. 5323	221.65	137.48
27. 5210	17.01	115.11	8. 5287	95.62	261.69
Jan. 2. 5177	202.20	219.61	9. 5437	87.27	274.17
51. 4614	202.20	194.12	10. 5272	83.28	275.42
52. 4582	272.58	213.63	12. 4733	70.01	277.06
23. 5272	267.50	214.82	13. 5043	37.53	173.00
25. 5434	241.11	156.87	17. 6100	201.92	245.17
26. 4523	221.33	118.21	21. 5273	255.15	207.74
29. 5334	118.58	155.99	22. 4791	242.00	105.10
			29. 4584	17.40	240.87
Dec. 4. 6178	355.60	103.00			

In the first column, the time has been corrected for aberration.

In the second and third columns, the correction for refraction has been applied, and in addition the third column has been corrected for the change in refraction between the refraction of least angle and of distance.

§4. Comparison of the stars with Standard Stars

In computing the tabular places, the following elements were applied, referred to the mean equinox of the date.

Star	Oct. 0.0, 1875	Nov. 0.0, 1877
α	$63^{\circ} 13'.4$	$339^{\circ} 39'.2$
δ	$1^{\circ} 4' 1.0$	$150 43.2$
$\log p$	$-1^{\circ} 42.1 (+5.5)$	$21 42.3 (+11.1)$
$\log q$	$123 52.3$	$123 57.1$
$\log r$	$6 35.2$	$6 35.0$
$\log s$	0.1	0.1

Station	Nov. 0.0, 1879	Nov. 0.0, 1880
ϵ	$75^{\circ} 51.7$	$184^{\circ} 2.5$
ρ	86 21.7	64 2.5
Δ	$-25.541(t-1879.63)$	$-22.5.3(t-1880.0)$
λ	124 1.8	124 4.1
μ	6 34.7	6 32.1
ν	0.08	0.08

Station	Jan. 0.0, 1884	Jan. 0.0, 1885
ϵ	$298^{\circ} 50.0$	$0^{\circ} 30.0$
ρ	357 34.5	338 44.2
Δ	$-19.29.7(t-1884.0)$	$-18.15.8(t-1885.0)$
λ	124 11.8	124 14.1
μ	6 34.3	6 34.2
ν	0.1	0.1

$$\Delta = 214''.00$$

$$\mu = 16^{\circ}.91988$$

$$\rho_0 = 9.53887$$

ρ_0 is the mean value for the total horizontal distance.

Computation of Capital

Year	Y	Y	Y	Y	Y	Y	Y
1895							
29.6026	172.83	173.05	3.02	0.52	45.54	46.55	0.77 + 0.60
31.5754	105.15	110.70	5.61	0.03	66.24	66.57	0.35 + 0.03
26.65863	215.80	211.47	4.31	0.03	9.03	8.02	1.01 + 0.71
7.0121	224.77	224.10	0.13	0.07	26.04	27.74	1.40 + 0.15
11.5778	143.47	143.23	1.76	0.75	52.12	54.20	1.50 + 0.20
17.2503	177.58	177.73	0.77	0.51	2.37	3.10	0.50 + 0.05
20.5578	167.20	166.45	0.75	1.25	54.04	54.00	0.04 + 0.30
29.5059	234.16	233.45	1.11	0.07	10.51	10.46	1.05 + 1.20
Oct. 2.5196	138.40	137.17	1.21	1.09	52.72	52.73	0.01 + 1.58
3.4777	102.76	102.00	1.22	0.07	55.50	55.57	0.07 + 0.08
17.4712	138.10	141.72	3.64	0.52	55.50	52.22	3.28 + 0.10
17.4863	207.76	211.27	1.51	0.70	6.57	1.90	4.01 + 0.52
20.4527	223.11	224.76	0.77	1.00	8.51	13.17	4.77 + 0.35
21.4717	225.38	222.22	3.16	0.75	24.52	28.41	3.87 + 0.01
22.4723	207.73	202.71	5.02	0.77	56.67	46.37	3.70 + 0.01
24.4254	125.03	116.85	8.20	0.54	0.77	53.77	3.00 + 1.50

Mean Time	x	x	u	Residual	x	y	u	Residual
1875								
Oct. 28. 4313	-132.89	-137.46	+ 4.57	+1.05	- 21.05	- 18.57	- 2.46	-1.46
29. 4305	-174.67	-177.72	+ 3.05	+0.10	1.57	1.61	- 0.04	0.41
1877								
Oct. 2. 5856	-225.58	-210.47	- 15.11	+0.10	13.40	11.52	+ 1.88	-1.85
12. 5438	151.84	122.57	+29.27	-1.23	2.60	2.73	- 0.13	-0.86
13. 5271	191.87	167.56	+22.61	0.10	- 4.37	- 4.10	- 0.27	0.53
14. 4962	215.20	201.47	+13.73	+0.75	- 11.24	- 10.52	- 0.72	+0.13
14. 5334	215.23	202.38	+12.85	+1.32	- 11.20	- 10.76	- 0.44	-0.17
15. 5042	218.97	216.12	+ 2.85	+1.59	- 16.35	- 16.26	- 0.09	-0.19
Nov. 6. 4758	203.33	206.30	- 2.97	+0.69	- 19.04	- 19.18	+ 0.14	-0.04
27. 4330	201.57	201.47	+ 0.10	+0.66	- 17.03	- 17.23	+ 0.20	+0.16
29. 4266	137.94	148.38	-10.44	-0.61	- 19.26	- 21.27	+ 2.01	-0.71
30. 4241	11.23	94.38	-13.10	-1.44	- 18.39	- 19.64	+ 1.25	-1.00
Dec. 6. 4332	-220.58	-217.70	- 2.48	+0.52	15.10	16.38	- 1.28	+0.03
7. 4331	-221.74	-223.46	+ 1.52	+0.50	17.73	20.77	- 1.04	-0.24
8. 3993	-208.29	-213.37	+ 5.08	+0.67	22.33	23.54	- 1.21	+0.03
9. 4138	-179.23	-188.60	+ 9.35	+0.49	24.08	24.79	- 0.71	-0.26

(Greenwich) (Greenwich) (Greenwich)							
Greenwich	1	2	3	4	5	6	7
1877							
Dec. 14. 4613	81.82	63.57 + 18.25 - 0.74	1.31	1.31	1.31	1.31	1.31
15. 4408	130.57	113.65 + 16.92 - 1.70	2.53	2.53	2.53	2.53	2.53
1879							
Oct. 10. 5970	220.38	218.12 + 2.26 - 0.15	2.70	2.70	2.70	2.70	2.70
12. 6390	147.76	141.78 + 5.98 - 0.16	3.07	3.07	3.07	3.07	3.07
13. 5885	90.06	83.46 + 6.60 + 0.00	6.11	6.11	6.11	6.11	6.11
16. 6508	-117.39	-125.62 + 5.63 + 0.03	33.06	33.06	33.06	33.06	33.06
17. 5460	-247.12	-248.23 + 1.11 + 0.23	30.27	30.27	30.27	30.27	30.27
20. 5641	-260.09	-259.57 - 0.52 + 0.45	11.45	11.45	11.45	11.45	11.45
23. 5840	-190.05	-184.57 + 5.48 + 1.18	11.76	11.76	11.76	11.76	11.76
24. 5885	-132.81	-128.01 - 4.80 + 0.42	21.32	21.32	21.32	21.32	21.32
31. 5322	220.53	218.70 + 1.83 - 0.52	4.70	4.70	4.70	4.70	4.70
Nov. 3. 5766	100.44	95.61 + 4.83 + 1.11	21.72	21.72	21.72	21.72	21.72
10. 5175	-232.30	-253.71 + 1.11 - 0.36	13.45	13.45	13.45	13.45	13.45
11. 4879	-252.38	-251.33 - 1.05 - 0.05	6.20	6.20	6.20	6.20	6.20
12. 5185	-233.73	-230.56 - 3.17 + 0.15	3.06	3.06	3.06	3.06	3.06
13. 4597	-199.54	-196.07 - 3.47 - 0.03	7.44	7.44	7.44	7.44	7.44

Dimensions of the ...

...	...	...	...	...	...	...	...	...
1977								
1. 4. 032	141.2	144.74	3.72	- 0.00	- 7.30	18.2	+ 0.2	+ 0.75
21. 4. 27	215.03	214.57	+ 0.22	+ 0.03	- 1.57	1.07	+ 0.08	+ 0.57
37. 1. 23	114.75	110.23	+ 3.40	- 0.42	25.18	28.10	3.72	+ 0.3
30 - 303	220.77	223.35	+ 2.56	- 0.07	21.37	24.44	2.80	+ 0.10
1978								
11. 6. 214	7.77	3.82	- 2.15	- 0.21	61.60	62.32	- 1.22	- 0.71
11. 6. 249	75.94	73.61	- 2.33	- 0.27	37.18	62.11	- 2.73	+ 0.36
12. 6. 034	137.01	134.07	- 2.24	+ 0.4	54.25	57.15	- 3.10	+ 0.26
14. 5. 34	221.53	227.11	- 1.42	- 0.05	32.11	36.18	- 3.37	+ 0.07
15. 3. 678	252.29	251.12	- 1.17	- 0.00	17.19	21.74	- 2.55	+ 0.24
16. 3. 773	257.56	257.39	- 0.17	- 0.00	3.15	5.41	- 2.26	+ 0.48
23. 6. 153	133.24	129.26	+ 3.98	- 1.80	42.25	42.15	+ 3.66	- 0.23
24. 2. 37	117.00	112.15	+ 4.55	- 0.76	- 32.30	34.65	+ 2.35	+ 1.14
25. 5. 777	221.78	219.16	+ 2.62	+ 0.34	- 16.27	18.71	+ 2.62	+ 0.57
31. 5. 222	13.03	11.43	+ 1.60	+ 0.24	58.32	59.34	- 1.02	- 0.76
1. 5. 651	56.30	57.34	+ 0.34	+ 0.37	27.70	39.98	- 2.28	- 0.00
2. 5. 439	118.14	119.80	+ 1.66	- 0.22	53.71	56.41	- 2.50	+ 0.07

(Continued)							
Nov. 7. 5383	-247.99	-246.91	-1.08	-0.56	...	6.67	+6.61 -0.57
9. 5640	-173.23	-170.37	-2.86	+0.40	33.51	-24.71	+1.20 -0.08
11. 5609	-38.28	-35.80	-2.48	+0.84	40.50	-50.44	+1.88 +0.15
13. 5742	37.02	37.01	-5.66	+1.22	17.51	-20.77	+2.26 0.00
16. 5414	228.25	227.43	+0.82	-1.50	3.59	-6.00	+2.47 -0.77
17. 4985	224.00	220.02	+1.98	-1.15	10.51	-10.53	+0.02 +0.73
20. 5209	95.86	91.53	+4.33	+0.37	47.23	47.36	-2.13 +0.00
21. 5055	31.89	27.60	+4.29	+0.96	22.56	53.27	-17.71 -0.22
1983							
Dec. 1. 5633	269.64	271.59	-1.95	+0.96	33.12	35.25	-0.43 +0.54
2. 6172	248.03	250.89	-2.86	-0.38	62.44	63.32	-0.85 -0.08
3. 5778	211.14	212.10	-4.72	-0.41	13.30	14.07	-1.37 -0.58
4. 5117	91.87	101.17	-9.28	+0.57	100.27	111.17	-4.72 +1.00
6. 5625	25.39	35.74	-10.35	+0.14	108.27	112.75	-4.48 -0.37
9. 5607	-171.38	-163.37	-8.01	-0.75	59.85	67.22	-7.37 -0.21
10. 5577	-211.80	-206.11	-5.69	+0.40	29.50	37.37	-7.87 -0.10
17. 5212	48.83	82.80	+16.03	-0.45	89.77	93.18	+3.41 +0.25

Computation of (L)

L		L		L		L	
1	2	3	4	5	6	7	8
10.521	30.01	47.94	-13.33	+0.01	15.71	10.77	47.94
11.407	-217.58	-221.73	+7.15	+0.40	-27.80	20.11	11.407
1.010	-189.88	-177.02	-12.86	-0.57	40.70	30.45	10.12
22.000	-210.41	-200.77	-10.64	-0.31	7.02	21.00	-11.38
23.480	-210.75	-210.73	+0.02	+0.00	-21.11	10.72	10.11
25.5434	-137.35	-154.58	+17.23	+1.17	-75.77	-70.34	-5.43
26.4523	-78.10	-100.28	+22.18	+0.25	10.54	-36.13	20.11
27.3331	30.01	110.71	+80.70	+0.69	-14.02	-12.50	+3.47
28.1110	-5.00	-0.11	+0.35	+0.00	-1.00	10.77	20.11
7.7174	-177.83	-161.77	+16.06	-0.00	30.11	20.11	+0.11
10.521	-222.03	-222.00	-0.03	-0.00	-1.00	-250	+2.76
12.714	-222.03	-222.83	+0.80	+0.58	-27.57	27.53	+0.14
15.5784	137.02	135.29	+1.73	-0.48	-20.24	-20.11	-0.11
16.500	280.45	277.18	+3.27	+0.26	10.11	10.11	+0.11
26.5105	-62.51	-70.30	+7.79	+0.53	10.11	10.11	+0.11
27.3331	30.01	110.71	+80.70	+0.69	-14.02	-12.50	+3.47



Equation of Condition (Continued)

Station	L	λ	μ	ν	ξ	η	ζ	δ
Jan. 8. 5287	260.43	257.11 + 5.07	2.74	- 32.13	2.10	- 3.87	10.77	
1. 5437	274.11	272.41 + 2.11 + 2.11		2.40	1.45	2.07	10.77	
10. 5272	273.52	268.94 + 4.58 + 0.11		30.23	27.11	2.33	10.77	
12. 4723	220.42	211.61 + 8.81 - 0.84			2.40	0.28 + 2.11		
13. 5043	166.43	156.97 + 9.46 + 1.00		27.11	22.21	4.41	10.77	
17. 6180	-123.13	- 7.27 - 7.4 - 0.11		2.11	14.55 + 2.11	1.87		
21. 5273	-202.35	-202.92 + 0.57 + 0.13	- 53.65	20.20 + 2.55 + 0.34				
22. 4991	-162.10	-163.42 + 1.02 - 0.62	- 77.26	- 32.31 + 3.05 - 0.33				
27. 4584	244.43	243.77 - 1.06 - 0.27	- 31.11	- 27.65	2.11	1.21		

§ 5. Equation of Condition.

In the equation of condition, we have employed seven variables, the correction to the mean longitude being separated into two parts, thus:

$$d\bar{O} = d\bar{O} + t d\mu.$$

The form of the equation arising from the ^{differences} ~~residuals~~

in x , is

$$\underline{a}(d\delta + t.l.\mu) + \underline{b} \cdot u d\tilde{T} + \underline{c} \cdot d\epsilon + \underline{d} \cdot d\Delta + \underline{e} \cdot \sin f \cdot d\psi + \underline{f} \cdot d\psi = 0$$

$$\underline{a} = \frac{f'}{\cos \varphi} \left\{ \cos(\tilde{T} + u) + \epsilon \cos(\tilde{T} + u) \right\}$$

$$\underline{b} = -\frac{f'}{\cos \varphi} \left\{ \cos(\tilde{T} + u) \left(\cos \epsilon \cos \varphi + \frac{\epsilon}{1 + \cos \varphi} \right) + \cos(\tilde{T} + \psi) \right\}$$

$$\underline{c} = \frac{f'}{\cos \varphi} \left\{ \cos(\tilde{T} + u) \left(\epsilon - \cos(\tilde{T} + \psi) \cos \varphi \right) \right\}$$

$$\underline{d} = \frac{\epsilon}{\Delta}$$

$$\underline{e} = -f' \tan \frac{1}{2} i \cdot r \cos(\tilde{T} + u) \cos u - \frac{p_0}{p} r \cos f \cos \psi$$

$$\underline{f} = f' \tan \frac{1}{2} i \cdot r \cos(\tilde{T} + u) \sin u + \frac{p_0}{p} r \cos f \sin \psi$$

in u , is

$$f' = \frac{p_0}{p} \sin f$$

and $\tilde{T} = \tilde{T} + u - u$

The second result in y is obtained by replacing f , F , and ξ , by g , G , and η , respectively.

In order to facilitate the solution of the normal equations certain transformations were made in the constant of condition for the different observations, as follows:

1175	1177	1178
$x = 2dE$	$x = 2dE$	$x = 2dE$
$y = 4ndu$	$y = 6ndu$	$y = 30d\mu$
$z = 4nd''$	$z = 4nd''$	$z = 20d''$
$u = 3d\epsilon$	$u = 4d\epsilon$	$u = 4d\epsilon$
$v = 3dL$	$v = 3dL$	$v = 3dL$
$w = 2 \sin \frac{1}{2} d$	$w = 2 \sin \frac{1}{2} d$	$w = 2 \sin \frac{1}{2} d$
$t = 2d\gamma$	$t = 3d\gamma$	$t = 2d\gamma$

1882	1883-4	1884-5
$x = 2d\mathcal{E}$	$x = 2d\mathcal{E}$	$x = 3d\mathcal{E}$
$y = 3d\mu$	$y = 45d\mu$	$y = 45d\mu$
$z = 4d\mathcal{P}$	$z = 3cd\mathcal{P}$	$z = 3cd\mathcal{P}$
$u = 3de$	$u = 4de$	$u = 4de$
$v = 2d\Delta$	$v = 2d\Delta$	$v = 3d\Delta$
$w = 2d\eta$	$w = 2d\eta$	$w = 2d\eta$
$t = 2d\mathcal{J}$	$t = 2d\mathcal{J}$	$t = 2d\mathcal{J}$

The transformed equation is the

$$a'x + d'y + b'z + c'u + d'v + e'w + f't = n$$

where the coefficients have the values given in the following table.

The line at each date contains the coefficients of the equation resulting from the reduction of the equation of the original time, and the residual n .

Quantities of Condition.

Year	Month	Day	1	2	3	4	5	6	7
1875									
Aug.	26.	6672	-0.279	+0.425	+0.335	+0.114	-0.294	+0.071	-0.090
			+0.147	-0.225	-0.119	+0.094	-0.102	-0.160	+0.496
	29.	6026	+0.222	-0.310	+0.202	+0.367	-0.305	-0.089	+0.010
			+0.091	-0.127	-0.005	+0.112	+0.073	+0.383	+0.339
	31.	5754	+0.430	-0.562	+0.315	+0.478	-0.173	-0.145	+0.079
			+0.019	-0.025	-0.056	+0.055	+0.094	+0.536	-0.000
Sept.	6.	5863	+0.233	-0.243	+0.340	+0.100	+0.330	-0.064	+0.114
			-0.125	+0.130	-0.114	+0.095	+0.012	+0.134	-0.578
	9.	6188	-0.178	+0.162	+0.215	+0.376	+0.350	+0.057	+0.025
			-0.097	+0.088	-0.066	+0.121	-0.000	-0.000	-0.454
	11.	5755	-0.444	+0.363	+0.308	+0.436	+0.226	+0.117	-0.046
			-0.030	+0.025	-0.049	+0.077	-0.084	-0.454	-0.190
	19.	5003	+0.175	-0.082	+0.192	+0.331	-0.312	-0.076	-0.003
			+0.104	-0.049	-0.071	+0.112	+0.067	+0.331	+0.374
	20.	5578	+0.314	-0.132	+0.221	+0.411	-0.259	-0.113	+0.036
			+0.067	-0.028	-0.057	+0.089	+0.085	+0.445	+0.250
	21.	5007	+0.013	0.000	+0.235	+0.151	+0.064	+0.003	+0.000

	x'	y'	z'	x''	y''	z''	x'''	y'''	z'''
1.4716	-0.121	+0.003	-0.091	+0.125	-0.029	-0.093	-0.5744	+1.62	
2.4716	-0.003	-0.045	+0.277	+0.42	+0.245	+0.108	-0.034	+1.21	
3.4716	-0.003	-0.005	-0.048	+0.087	-0.032	-0.422	-0.238	+0.01	
4.4716	-0.047	-0.077	+0.356	+0.436	+0.159	+0.126	-0.065	+1.82	
5.4716	-0.003	0.000	-0.053	+0.060	-0.086	-0.464	-0.087	+0.01	
6.4716	+0.366	+0.204	+0.440	+0.153	+0.221	-0.112	+0.128	-3.54	
7.4716	-0.107	-0.083	-0.112	+0.059	+0.050	+0.331	-0.465	+3.58	
8.4716	+0.171	+0.141	+0.200	+0.092	+0.329	-0.047	+0.100	-1.51	
9.4716	-0.125	-0.100	-0.107	+0.106	+0.003	+0.085	-0.551	+4.61	
10.4716	+0.055	+0.000	-0.200	-0.000	-0.000	-0.000	-0.000	-0.000	
11.4716	-0.122	-0.111	-0.095	+0.122	-0.021	-0.045	-0.530	+4.98	
12.4716	-0.087	-0.083	+0.208	+0.203	+0.346	+0.031	+0.044	+3.16	
13.4716	-0.107	-0.104	-0.074	+0.120	-0.044	-0.000	-0.400	+3.17	
14.4716	-0.222	-0.222	+0.212	+0.294	+0.316	+0.066	+0.011	+5.62	
15.4716	-0.085	-0.086	-0.057	+0.117	-0.063	-0.274	-0.397	+3.90	
16.4716	-0.455	-0.494	+0.320	+0.415	+0.102	+0.117	-0.054	+3.20	
17.4716	-0.216	-0.010	-0.240	+0.000	-0.000	-0.000	-0.130	+3.00	

Oct. 21 1877	-0.392	-0.475	+0.392	+0.171	-0.212	-0.006	-0.012	+1.571
	+0.136	+0.172	-0.117	+0.071	-0.052	-0.017	+0.011	-2.00
29.4305	-0.245	-0.320	+0.304	+0.100	-0.217	+0.056	-0.015	+3.55
	+0.145	+0.172	-0.115	+0.072	-0.051	-0.062	+0.460	-3.96
1877								
Oct. 2. 5836	-0.285	+0.270	+0.257	-0.322	-0.323	+0.055	+0.001	-15.11
	+0.051	-0.048	-0.010	+0.050	+0.018	+0.156	+0.353	+1.88
12.5438	+0.420	-0.258	+0.135	-0.370	+0.191	-0.070	+0.026	+29.27
	-0.055	+0.034	+0.002	+0.053	+0.004	+0.165	-0.363	-0.13
13.5271	+0.325	-0.190	+0.204	-0.335	+0.264	-0.004	+0.013	+22.61
	-0.054	+0.032	-0.006	+0.055	-0.006	+0.012	-0.317	-0.21
14.4962	+0.176	-0.108	+0.191	-0.268	+0.314	-0.053	-0.001	+13.73
	-0.045	+0.025	-0.002	+0.052	-0.006	-0.135	-0.312	-0.72
14.5347	+0.107	-0.105	+0.071	-0.265	+0.015	-0.053	-0.002	+12.05
	-0.045	+0.025	-0.002	+0.052	-0.017	-0.145	-0.340	-0.04
15.5042	+0.031	-0.010	+0.003	-0.103	+0.037	-0.007	-0.015	+2.05
	-0.037	+0.020	+0.002	+0.104	-0.025	-0.007	-0.000	-0.07

Continued

17. 4233	-0.077	-0.021	+0.507	-0.021	-0.021	-0.22	-0.024	-2.95
18. 4233	-0.020	-0.020	+0.014	+0.039	-0.030	-0.05	-0.011	+0.14
27. 4330	-0.072	-0.038	+0.205	-0.147	+0.314	-0.027	-0.020	+0.10
28. 4330	-0.023	-0.020	+0.005	+0.040	-0.027	-0.309	-0.236	+0.20
29. 4266	-0.365	-0.358	+0.370	-0.022	+0.231	-0.038	-0.038	-10.44
30. 4241	+0.026	+0.025	-0.008	-0.026	-0.038	-0.448	-0.041	+2.01
30. 4241	-0.480	-0.487	+0.432	-0.171	+0.147	+0.030	-0.040	-13.10
31. 4222	+0.022	+0.022	-0.018	+0.028	-0.031	-0.442	+0.069	+1.25
32. 4330	-0.104	-0.101	+0.188	-0.182	-0.339	+0.038	+0.015	-2.40
33. 4331	+0.041	+0.047	-0.001	+0.044	+0.020	+0.211	+0.289	-1.28
34. 4331	+0.011	+0.022	+0.216	-0.125	-0.041	+0.025	+0.027	+1.52
35. 4331	+0.029	+0.027	+0.001	+0.037	+0.032	+0.391	+0.233	-1.04
36. 4331	+0.130	+0.174	+0.200	-0.096	-0.333	+0.008	+0.036	+0.20
37. 4331	+0.017	+0.022	-0.001	+0.030	+0.037	+0.472	+0.163	-1.21
38. 4130	+0.246	+0.323	+0.325	-0.077	+0.274	-0.009	+0.043	+9.35
39. 4130	+0.004	+0.005	-0.006	+0.026	+0.039	+0.525	+0.078	-0.71
40. 4613	+0.431	+0.639	+0.004	-0.050	+0.077	-0.005	+0.005	+18.25

Comparison of Conditions (Continued)

Station	1	2	3	4	5	6	7	8
1878	-0.075	-0.076	-0.078	+0.055	+0.052	+0.067	-0.071	+0.071
10.5940	-0.077	+0.051	+0.231	-0.047	+0.177	-0.002	+0.022	+16.72
	-0.050	-0.076	-0.012	+0.050	+0.003	+0.141	-0.327	+0.66
1879								
Oct 10. 5970	-0.122	+0.167	-0.006	-0.303	+0.346	-0.001	-0.029	+2.50
	+0.091	-0.124	-0.153	+0.033	-0.033	-0.332	-0.061	+1.07
12.0510	-0.437	+0.537	+0.077	-0.177	+0.221	+0.030	-0.004	+5.07
	+0.075	-0.091	-0.109	+0.051	+0.032	-0.317	+0.284	-2.31
12.5913	-0.520	+0.001	-0.003	-0.534	+0.131	+0.147	-0.077	+0.00
	+0.057	-0.066	-0.083	+0.046	+0.044	-0.254	+0.414	-2.08
16.6508	-0.482	+0.461	-0.423	-0.463	-0.195	+0.062	-0.065	+5.07
	-0.012	+0.012	-0.083	+0.014	+0.057	+0.051	+0.579	-3.54
19.5460	-0.160	+0.123	-0.293	-0.028	-0.387	+0.035	-0.015	+1.11
	-0.062	+0.047	-0.136	+0.018	+0.034	+0.316	+0.375	-1.56
20.5641	-0.017	+0.012	-0.156	-0.271	-0.404	+0.020	+0.006	-0.52
	-0.072	+0.050	-0.143	+0.029	+0.020	+0.372	+0.243	-1.70
23.5340	+0.391	-0.193	+0.047	-0.422	-0.288	-0.027	+0.001	-5.07

Continuation of Computations (Continued)

Station	1	2	3	4	5	6	7	8
10. 24.5884	-0.005	+0.074	-0.017	+0.052	-0.023	+0.075	-0.008	+0.056
	+0.1471	-0.211	-0.034	-0.506	-0.200	-0.072	+0.070	4.00
	0.056	+0.024	-0.082	+0.051	-0.036	+0.314	-0.341	+1.335
31. 3.322	-0.072	-0.003	-0.041	-0.286	+0.341	-0.006	+0.007	+1.03
	+0.083	+0.003	-0.114	+0.034	-0.009	-0.338	-0.118	+0.003
10. 3.0766	-0.001	-0.111	+0.026	-0.5717	+0.149	+0.045	-0.070	+4.03
	+0.057	+0.014	-0.012	+0.047	+0.037	-0.272	+0.373	-2.11
10. 5175	-0.011	-0.043	-0.137	-0.265	-0.395	+0.024	+0.002	+1.21
	-0.063	-0.044	-0.107	+0.055	+0.024	+0.347	+0.080	-1.00
11. 4877	+0.035	+0.065	+0.063	-0.283	-0.372	+0.037	+0.022	-1.05
	-0.067	-0.053	-0.127	+0.035	+0.011	+0.384	+0.150	-1.02
12. 5155	+0.226	+0.187	+0.000	-0.236	-0.359	-0.008	+0.040	-3.17
	-0.071	-0.027	-0.117	+0.045	-0.003	+0.376	-0.005	-1.14
13. 4577	+0.045	+0.310	+0.053	-0.404	-0.300	-0.022	+0.053	-3.47
	-0.067	-0.060	-0.102	+0.050	0.010	-0.000	-0.145	+0.67
14. 4622	+0.401	+0.435	-0.551	-0.476	-0.226	-0.037	+0.006	-7.71
	-0.051	-0.006	-0.081	+0.050	-0.028	+0.334	-0.283	+0.52

Continued (Continued)

2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}
1.4727	-0.017	-0.027	-0.047	-0.268	+0.335	-0.012	-0.013 + 0.82
	+0.075	+0.108	-0.132	+0.030	0.044	-0.317	-0.169 + 0.08
2.4305	-0.333	-0.064	-0.421	-0.244	0.293	+0.051	-0.040 + 3.48
	-0.030	-0.077	-0.107	+0.012	+0.045	+0.167	+0.310 - 2.73
3.4307	-0.223	-0.452	-0.347	-0.207	-0.340	+0.041	-0.023 + 2.50
	-0.045	-0.071	-0.100	+0.013	+0.037	+0.232	+0.432 - 2.00
4.4314	-0.510	+0.070	-0.414	-0.512	-0.014	+0.104	-0.015 - 2.15
	+0.017	-0.010	-0.040	+0.030	+0.147	-0.078	+0.584 - 1.22
11.0267	-0.517	+0.005	-0.450	-0.410	-0.172	+0.103	-0.092 - 2.33
	-0.023	+0.027	-0.054	+0.030	+0.147	+0.072	+0.505 - 2.03
12.0034	0.460	+0.564	0.440	0.307	-0.314	+0.070	-0.683 - 2.34
	-0.055	+0.068	-0.071	+0.006	+0.133	+0.232	+0.546 - 3.10
14.5834	-0.250	+0.282	-0.347	-0.180	-0.530	+0.065	-0.040 - 1.42
	-0.107	+0.117	-0.072	+0.101	+0.084	+0.481	+0.301 - 3.37
15.3678	-0.124	+0.128	-0.278	-0.170	-0.586	+0.037	-0.025 - 1.17
	0.123	+0.120	-0.071	+0.114	+0.051	+0.200	+0.228 - 2.52

Continuation of Computations

Station	1'	2'	3'	4'	5'	6'	7'	8'
Oct. 16.5773	+0.027	-0.026	-0.218	-0.245	-0.601	+0.012	-0.001	0.00
	-0.130	+0.125	-0.081	+0.115	+0.013	+0.000	-0.006	0.00
23.6158	+0.505	-0.249	-0.458	-0.284	+0.302	-0.000	+0.000	+5.99
	+0.075	-0.037	-0.076	+0.063	-0.107	-0.000	-0.000	+3.00
24.5637	+0.372	-0.160	-0.387	-0.207	+0.427	-0.001	+0.000	+4.00
	+0.109	-0.047	-0.092	+0.090	-0.081	-0.000	-0.000	+3.00
25.5777	+0.192	-0.070	-0.287	-0.199	+0.512	-0.000	+0.001	+2.00
	+0.133	-0.048	-0.097	+0.113	-0.044	-0.000	-0.000	+2.00
31.5522	-0.538	-0.020	-0.397	-0.532	+0.027	+0.101	-0.000	+1.00
	+0.024	+0.001	-0.034	+0.030	+0.138	-0.000	+0.000	+1.00
Nov. 1.5651	-0.524	-0.055	-0.442	-0.444	-0.134	+0.004	-0.001	+0.00
	-0.011	-0.001	-0.047	+0.065	+0.140	+0.027	+0.570	-2.28
2.2437	-0.475	-0.001	-0.444	-0.343	-0.280	+0.098	-0.000	+1.00
	-0.043	-0.007	-0.062	+0.062	+0.132	+0.000	+0.000	+2.00
7.5383	+0.146	+0.074	-0.191	-0.318	-0.577	-0.000	+0.000	+1.00
	-0.124	-0.062	-0.064	+0.148	-0.014	+0.570	-0.500	+0.00
12.2640	+0.435	+0.278	-0.545	-0.517	-0.398	-0.000	+0.000	+2.00

Continuation of Table (1)

	b'	c'	d'	e'	f'	g'	h'	i'
	-0.094	-0.060	-0.031	+0.136	-0.081	+0.471	-0.325	+1.20
Nov. 11. 5609	+0.596	+0.459	-0.415	-0.524	-0.084	-0.000	+0.081	-2.48
	+0.026	+0.020	-0.026	+0.085	-0.118	+0.184	-0.495	+1.88
12. 5542	+0.593	+0.496	-0.468	-0.431	+0.093	-0.090	+0.079	-2.06
	-0.004	-0.004	-0.033	+0.073	-0.119	+0.011	-0.514	+3.36
16. 5414	+0.045	+0.050	-0.216	-0.244	+0.531	-0.014	-0.001	+0.82
	+0.133	+0.146	-0.083	+0.128	-0.014	-0.526	-0.002	+2.47
17. 4985	+0.129	+0.151	-0.169	-0.344	+0.518	+0.015	-0.026	+1.98
	+0.131	+0.153	-0.070	+0.142	+0.023	-0.546	+0.070	+0.18
20. 5209	-0.490	-0.670	-0.301	-0.570	+0.214	+0.086	-0.083	+4.33
	+0.064	+0.088	-0.027	+0.104	+0.115	-0.320	+0.482	-2.13
21. 5055	-0.526	-0.754	-0.374	-0.538	+0.064	+0.097	-0.089	+4.29
	+0.032	+0.046	-0.029	+0.082	+0.129	-0.182	+0.551	-1.71
1883								
Dec. 1. 5633	-0.081	+0.106	-0.234	+0.250	+0.634	+0.049	-0.045	-1.95
	+0.220	-0.288	+0.287	+0.103	+0.083	-0.487	+0.429	-0.43
2. 6172	-0.226	+0.285	-0.329	+0.264	+0.586	+0.069	-0.079	-2.06

Equations of Condition (Continued)

Year, month day	a_0'	a_1'	b_1'	c_1'	d_1'	e_1'	f_1'	n
1883	+0.193	-0.243	+0.277	+0.069	+0.148	-0.372	+0.525	-0.88
Dec. 3. 5798	-0.344	+0.419	-0.395	+0.311	+0.504	+0.083	-0.106	-4.72
	+0.155	-0.189	+0.249	+0.046	+0.198	-0.243	+0.580	-1.37
5. 6167	-0.522	+0.589	-0.404	+0.466	+0.236	+0.096	-0.138	-9.25
	+0.044	-0.050	+0.169	+0.044	+0.260	+0.068	+0.576	-4.92
6. 5625	-0.560	+0.608	-0.337	+0.522	+0.083	+0.092	-0.140	-10.36
	-0.018	+0.020	+0.142	+0.066	+0.263	+0.212	+0.517	-4.48
9. 5607	-0.410	+0.391	-0.048	+0.466	-0.381	+0.042	-0.083	-8.01
	-0.212	+0.202	+0.195	+0.157	+0.157	+0.520	+0.124	-7.37
10. 5577	-0.259	+0.235	-0.041	+0.373	-0.481	+0.015	-0.046	-5.69
	-0.255	+0.232	+0.248	+0.157	+0.087	+0.536	-0.046	-7.87
17. 5212	+0.579	-0.347	-0.266	+0.533	+0.194	-0.069	+0.115	+16.03
	+0.091	-0.055	+0.121	+0.122	-0.218	-0.411	-0.320	+3.41
27. 5298	-0.546	+0.084	-0.356	+0.501	+0.117	+0.094	-0.137	-13.33
	-0.002	0.000	+0.144	+0.059	+0.259	+0.163	+0.532	-4.93
1884								
Jan. 2. 4879	+0.086	+0.010	-0.211	+0.249	-0.518	-0.033	+0.025	+4.15
	-0.260	0.029	+0.276	+0.124	-0.144	+0.425	-0.303	-8.99

Continued

<i>Greenwich Mean Time</i>	<i>a</i>	<i>b</i>	<i>b'</i>	<i>c'</i>	<i>d'</i>	<i>e'</i>	<i>f</i>	<i>g</i>
Jan. 21. 4804	-0.322	-0.308	-0.035	+0.400	-0.418	+0.029	-0.062	-10.86
	-0.223	-0.213	+0.210	+0.151	+0.118	+0.507	+0.056	-10.12
22. 4502	-0.162	-0.162	-0.065	+0.313	-0.488	+0.003	-0.026	-4.63
	-0.252	-0.252	+0.257	+0.140	+0.049	+0.503	-0.102	-11.38
23. 4272	+0.027	+0.028	-0.165	+0.250	-0.507	-0.024	+0.013	+3.18
	-0.256	-0.267	+0.285	+0.110	-0.025	+0.445	-0.250	-10.69
25. 5434	+0.413	+0.469	-0.423	+0.305	-0.361	-0.069	+0.086	+17.23
	-0.171	-0.194	+0.234	+0.042	-0.164	+0.170	-0.456	-6.45
26. 4523	+0.520	+0.611	-0.453	+0.386	-0.234	-0.078	+0.106	+22.18
	-0.105	-0.123	+0.183	+0.040	0.201	+0.015	-0.473	-2.61
29. 5334	+0.506	+0.664	-0.187	+0.490	+0.273	-0.107	+0.099	+20.17
	+0.119	+0.156	+0.125	+0.132	-0.188	+0.395	-0.248	+5.90
Dec. 4. 6198	-0.401	+0.705	+0.209	+0.511	0.026	+0.012	+0.130	+8.35
	-0.059	+0.104	+0.193	+0.030	+0.166	+0.403	+0.318	-2.29
7. 5974	-0.192	+0.300	+0.249	+0.332	-0.314	+0.001	-0.030	+3.06
	-0.186	+0.291	+0.347	+0.038	+0.051	+0.495	-0.188	+2.20

Orbit of Comet (continued)

	a	a'	b'	c	d	e	f	g
Dec. 2.5828	0.058	+0.386	+0.177	+0.220	0.351	0.022	+0.010	+2.18
	-0.116	+0.292	+0.364	-0.003	0.016	+0.415	-0.538	+2.76
Jan. 5.5794	+0.000	0.120	-0.026	+0.315	-0.347	0.004	+0.057	+0.80
	0.182	+0.260	+0.339	-0.042	-0.002	+0.255	-0.430	+2.14
15.5784	+0.335	-0.344	+0.237	+0.502	+0.211	-0.044	+0.106	+1.73
	+0.096	-0.099	+0.237	+0.071	-0.114	-0.556	-0.106	0.221
19.5685	+0.010	-0.008	+0.075	+0.290	+0.432	+0.034	0.031	+3.27
	+0.159	-0.121	+0.343	-0.015	+0.024	0.478	+0.114	-5.32
26.5105	-0.382	+0.114	+0.208	+0.534	-0.107	+0.053	-0.114	+7.77
1885-	-0.096	+0.029	+0.221	+0.052	+0.015	+0.459	+0.220	+0.02
Jan. 2.5323	+0.356	+0.060	-0.084	+0.545	-0.145	-0.074	+0.133	+0.36
	-0.059	-0.010	+0.143	-0.027	0.165	-0.152	-0.413	+1.57
Jan. 5.5287	+0.123	+0.070	+0.150	+0.308	+0.402	+0.010	+0.016	+2.47
	+0.155	+0.088	+0.304	+0.030	-0.034	-0.572	+0.283	3.83
Jan. 5.437	+0.029	+0.018	+0.087	+0.286	+0.424	+0.030	-0.021	+2.38
	+0.158	+0.100	+0.341	+0.006	+0.013	0.500	+0.105	5.07
10.5272	-0.065	-0.045	+0.002	+0.002	+0.417	+0.048	-0.026	+4.58

§ 6. Normal Equations, and their Solution.

The equations of condition at each opposition have been solved separately, giving rise thereby to the following six sets of Normal Equations.

1 75

$$\begin{aligned}
 &1.38131 \quad 0.40747 \quad -0.45532 \quad -0.27111 \quad -0.35403 \quad -0.02217 \quad -0.03857 \quad -0.03857 \\
 &-0.44014 \quad +1.7435 \quad +1.2571 \quad +0.4719 \quad +0.7523 \quad +0.0772 \quad -0.0151 \quad +0.0102 \\
 &-0.0056 \quad +0.0056 \quad +1.7110 \quad +0.2177 \quad -0.2426 \quad -0.0376 \quad -0.0376 \quad +1.1712 \\
 &-0.2910 \quad +0.4449 \quad +0.3855 \quad +1.5771 \quad +0.5077 \quad +0.3706 \quad +0.2780 \quad +1.5775 \\
 &-0.3540 \quad +0.0723 \quad -0.0237 \quad +0.5077 \quad +2.0654 \quad +0.5733 \quad -0.0845 \quad -2.3766 \\
 &1.0122 \quad +0.2177 \quad -0.2426 \quad +0.3706 \quad +0.5733 \quad +3.2005 \quad +0.1275 \quad -17.6476 \\
 &0.0385 \quad -0.4415 \quad -0.6475 \quad +0.2780 \quad -0.0845 \quad +0.1275 \quad +1.8552 \quad -20.7268
 \end{aligned}$$

$$[nn] = 373.61$$

1377.

limb - 3005.14

1899

W.C. = 320.72

1550

$$3.6757x + 0.6603y + 0.7934u + 0.6995v - 1.2519w + 0.3929t + 0.2330y - + 1.1428$$

$$0.6603 + 2.6925 + 2.5184 + 0.2997 - 0.2273 + 0.0409 - 0.8237 = -3.2772$$

$$0.7934 + 2.5184 + 3.2342 + 0.3575 - 0.1474 + 0.3166 - 0.5513 = -3.5201$$

$$0.6995 + 0.2997 + 0.3575 + 3.0654 - 0.1079 + 0.9771 - 0.8348 = +7.5177$$

$$-1.2519 - 0.2273 - 0.1474 - 0.1079 + 2.8964 + 0.2408 + 0.0178 = -7.9406$$

$$0.3929 + 0.0409 + 0.3166 + 0.9771 + 0.2408 + 3.7894 + 0.2188 = +17.9767$$

$$0.2330 - 0.8237 - 0.5513 - 0.8348 + 0.0178 + 0.2488 + 3.2022 = -17.9708$$

$$[E_{11}] = 260.12$$

1551

$$-0.9966x - 0.0459y - 0.6885u - 0.1592v - 1.3494w + 1.0744t - 0.3561y = +83.314$$

$$-0.0459 + 2.0812 - 1.1794 - 0.0962 + 0.4673 + 0.3333 - 1.6326 = -27.8641$$

$$-0.6885 - 1.1794 + 2.6473 - 0.1540 + 0.4345 - 0.1376 + 1.4021 = -4.3953$$

$$-0.1572 - 0.0962 - 0.1540 + 3.1612 + 0.2948 + 0.8543 + 0.5295 = -11.357$$

$$-1.3494 + 1.0744 - 0.3561 + 2.5739 - 0.2877 = +12.3036$$

$$1.0744 + 0.3333 - 0.1376 + 0.8543 - 0.6802 + 2.5739 - 0.2877 = +12.3036$$

$$-0.9966 - 0.0459 - 0.6885 - 0.1592 - 1.3494 + 1.0744 + 0.3333 - 0.1376 + 0.8543 - 0.6802 + 2.5739 - 0.2877 = -11.357$$

$$[E_{11}] = 2818.69$$

1.224 5-

$-0.2977 + 1.8648 + 0.5662 + 0.0839 - 0.1725 + 0.0000 + 0.0000 + 0.0000$
 $+ 0.0000 + 0.0000 + 0.0000 + 0.0723 + 0.0000 + 0.0000 + 0.0000$
 $+ 0.0000 + 0.0000 + 0.2041 + 1.8123 + 0.0000 + 0.3470 - 0.2384 = +3.3694$
 $+ 0.0000 + 0.0000 + 0.0723 + 0.3572 + 0.0000 - 1.0815 + 0.0410 = +17.2571$
 $+ 0.0000 + 0.0000 - 0.0723 + 0.3470 + 0.0000 + 0.0000 + 0.0000 + 0.0000$
 $+ 0.0000 + 0.0000 + 0.3748 - 0.2384 + 0.0000 + 0.1973 + 1.5099 - 0.6289$

In solving the foregoing equations the values of
 the constants are obtained in terms of the
 elements of each set, the sum of the squares of
 the residuals is then

1575

$$v = 2 d \epsilon = +0.2306 + 0.0856 y$$

$$g = 2 d \delta = +1.1308 + 0.1035 y$$

$$z = 2 d \zeta = +1.0627 - 0.2810 y$$

$$w = 2 d \eta = +0.2042 + 0.1371 y$$

$$t = 2 d \gamma = -5.4652 - 0.0423 y$$

$$y = 60 d \mu = -11.7033$$

1877

$$x = 2 d \epsilon = +41.5755 - 0.6784 y$$

$$g = 4 d \delta = +0.7711 - 0.2445 y$$

$$v = 2 d \Delta = +5.3676 + 0.4017 y$$

$$v = 2 \sin \gamma d \lambda = -0.9222 - 0.1257 y$$

$$t = 2 d \gamma = -6.7543 + 0.0633 y$$

$$y = 60 d \mu = -12.7927$$

1877

$$v = 2 d \epsilon = -0.1861 + 0.0856 y$$

$$g = 2 d \delta = -1.1308 + 0.1035 y$$

$$z = 2 d \zeta = -1.0627 + 0.2810 y$$

$$w = 2 d \eta = +1.0444 - 0.0494 y$$

$$t = 2 d \gamma = -2.3857 - 0.0174 y$$

$$y = 60 d \mu = -5.6361 - 0.0226 y$$

$$x = 2 d \epsilon = -0.7711 + 0.2445 y$$

$$g = 4 d \delta = -2.4076 + 0.3171 y$$

$$v = 2 d \Delta = +1.2084 - 0.2057 y$$

$$v = 2 d \Delta = +4.4636 + 0.3221 y$$

$$v = 2 \sin \gamma d \lambda = -2.4878 - 0.0278 y$$

$$t = 2 d \gamma = -5.7583 - 0.1178 y$$

$$y = 60 d \mu = -12.7927$$

1883-4

$$\begin{aligned}
 \epsilon &= 2d\epsilon & = +30''.1865 - 0.0710\mu \\
 \rho &= 3d\rho & = -12.0254 + 0.7412\mu \\
 \delta &= 4d\delta & = +0.2713 - 0.1682\mu \\
 \alpha & & = -1.0526 - 0.0616\mu \\
 \beta & & = +1.2749 - 0.2223\mu \\
 \gamma & & = -5.5131 - 0.0310\mu \\
 \delta & & = +6.8012\mu
 \end{aligned}$$

1884-5

$$\begin{aligned}
 \epsilon &= 3d\epsilon & = -9''.5267 - 0.0110\mu \\
 \rho &= 3d\rho & = -2.7017 - 0.1926\mu \\
 \delta &= 4d\delta & = +9.3776 - 0.1024\mu \\
 \alpha &= 3d\alpha & = +2.2474 + 0.1762\mu \\
 \beta &= 2d\beta & = -0.1375 - 0.0777\mu \\
 \gamma &= 2d\gamma & = -5.6351 - 0.1216\mu \\
 \delta &= 45d\delta & = -1.1739\mu
 \end{aligned}$$

From these we obtain the following corrections to the elements

1875

$$\begin{aligned}
 d\epsilon &= +0''.0309 + 1.926d\mu \\
 d\rho &= +0.7567 + 11.640\mu \\
 d\delta &= +0.2383 + 26.898\mu \\
 d\alpha &= -0.7316 - 0.962\mu \\
 d\beta &= +0''.3543 - 15.743\mu \\
 d\gamma &= +0.0075 + 0.088\mu \\
 d\mu &= -0''.06964
 \end{aligned}$$

1877

$$\begin{aligned}
 d\epsilon &= +5''.5657 - 20.953d\mu \\
 d\rho &= +0.5162 - 36.669\mu \\
 d\delta &= -1.0768 - 6.734\mu \\
 d\alpha &= -0.6028 + 1.267\mu \\
 d\beta &= +1''.9565 + 30.021\mu \\
 d\gamma &= -0.0127 - 0.055\mu \\
 d\mu &= 0''.05704
 \end{aligned}$$

1877.

$$d\tilde{Q} = -1^{\circ}.3705 + 3.036 d\mu$$

$$d\tilde{P} = +0.8964 - 114.229$$

$$d\tilde{M} = -1.8385 - 1.519$$

$$d\tilde{J} = -0.7545 - 0.339$$

$$d\tilde{\Delta} = +0''.3481 -$$

$$d\tilde{e} = -0.0013 + 0.042$$

$$d\mu = +0^{\circ}.0649$$

$$d\tilde{Q} = -0^{\circ}.0676 - 2.513 d\mu$$

$$d\tilde{P} = -2.3819 + 47.159$$

$$d\tilde{M} = -2.9078 - 3.643$$

$$d\tilde{J} = -0.7545 - 0.339$$

$$d\tilde{\Delta} = +0''.3481 -$$

$$d\tilde{e} = +0.0019 - 0.036$$

$$d\mu = -0^{\circ}.04994$$

$$d\tilde{Q} = +4^{\circ}.0410 - 0.022 d\mu$$

$$d\tilde{P} = -10.7322 + 111.174$$

$$d\tilde{M} = +1.107 - 4.174$$

$$d\tilde{J} = -0.7390 - 0.696$$

$$d\tilde{\Delta} = -0''.5263 - 5.173$$

$$d\tilde{e} = +0.0003 - 0.033$$

$$d\mu = +0^{\circ}.04047$$

$$d\tilde{Q} = -0^{\circ}.8502 - 0.165 d\mu$$

$$d\tilde{P} = -2.4113 - 28.890$$

$$d\tilde{M} = -0.1607 - 19.212$$

$$d\tilde{J} = -0.7544 - 2.736$$

$$d\tilde{\Delta} = +0''.7491 + 10.992$$

$$d\tilde{e} = +0.0110 - 0.021$$

$$d\mu = -0^{\circ}.00710$$

In the foregoing correction, the coefficient of $d\mu$ is chosen, that if $d\mu$ is expressed in degrees, each such correction will be expressed in the same units. Before applying the corrections to the assumed elements two sets were obtained, in which different values of $d\mu$ were used.

1st set, $d\mu = 0$

$a = 1.141 \pm 0.001$ $b = 1.141 \pm 0.001$

1st set

$$d\mu = 0 \quad 1.141 \pm 0.001$$

$$d\mu = 0 \quad 1.141 \pm 0.001$$

$$d\mu = +0.45.4 \pm 1.17.5$$

$$d\mu = -0.3.2 \pm 0.20.2$$

$$d\mu = +0.14.3 \pm 1.33.9$$

$$d\mu = -1.38.1 \pm 0.24.7$$

$$d\mu = -0.43.9 \pm 0.14.3$$

$$d\mu = -0.39.9 \pm 0.3.7$$

$$d\mu = +0.13.4 \pm 0.13.4$$

$$d\mu = +1.4.5 \pm 0.13.4$$

$$d\mu = +0.0075 \pm 0.0028$$

$$d\mu = +0.0013 \pm 0.0008$$

$$d\mu = 0$$

$$d\mu = 0$$

$$[m.m.] = 13.15$$

$$d\delta = +5^{\circ} 33'.7 \pm 0^{\circ} 11'.2$$

$$= +6^{\circ} 45'.6 \pm 0^{\circ} 6'.2$$

$$+0^{\circ} 31.0 \pm 2^{\circ} 18.6$$

$$= +2^{\circ} 36.5 \pm 0^{\circ} 50.0$$

$$-1^{\circ} 4.6 \pm 1^{\circ} 39.8$$

$$= -0^{\circ} 41.6 \pm 0^{\circ} 35.7$$

$$+1''.956 \pm 0''.534$$

$$= +0''.244 \pm 0''.210$$

$$-0.0127 \pm 0.0049$$

$$= -0.0075 \pm 0.0013$$

$$[m.v.] = 20.02$$

$$-1^{\circ} 23'.4 \pm 0^{\circ} 3'.2$$

$$= -1^{\circ} 22'.2 \pm 0^{\circ} 3'.2$$

$$+0^{\circ} 53.8 \pm 0^{\circ} 59.7$$

$$= +0^{\circ} 9.3 \pm 1^{\circ} 1.9$$

$$-1^{\circ} 10'.1 \pm 1^{\circ} 10'.1$$

$$= -1^{\circ} 10'.1 \pm 1^{\circ} 10'.1$$

$$-0^{\circ} 45.3 \pm 0^{\circ} 2.7$$

$$= -0^{\circ} 45.4 \pm 0^{\circ} 2.6$$

$$+0''.348 \pm 0''.142$$

$$= +0''.336 \pm 0''.139$$

$$-0.0013 \pm 0.0003$$

$$= -0.0013 \pm 0.0003$$

$$d\mu = 0$$

$$= +0^{\circ} 00.649 \pm 0^{\circ} 00.293$$

$$[m.v.] = 10.13$$

$$\Delta \alpha = -0^{\circ} 4'.2 \pm 0^{\circ} 5'.5$$

$$\Delta \delta = -22.7^{\circ} \pm 12.7^{\circ}$$

$$\Delta \mu_{\alpha} = -24.2^{\circ} \pm 11.1^{\circ}$$

$$\Delta \mu_{\delta} = -46.0^{\circ} \pm 2.4^{\circ}$$

$$\Delta \alpha = +2''.232 \pm 0''.349$$

$$\Delta \delta = +0.0019^{\circ} \pm 0.0019^{\circ}$$

$$\Delta \mu = 0$$

$$= +0^{\circ} 3'.3 \pm 0^{\circ} 5'.4$$

$$= -44.2^{\circ} \pm 30.8^{\circ}$$

$$= -24.2^{\circ} \pm 11.1^{\circ}$$

$$= -46.0^{\circ} \pm 2.4^{\circ}$$

$$= +1''.331 \pm 0''.161$$

$$= +0.0037^{\circ} \pm 0.0008^{\circ}$$

$$= -0^{\circ}.04774 \pm 0^{\circ}.00279$$

$$[\mu_{\alpha}] = 18.24$$

Table 1

$$\Delta \alpha = +4^{\circ} 5'.5 \pm 0^{\circ} 1'.1$$

$$\Delta \delta = -14.0^{\circ} \pm 22.3^{\circ}$$

$$\Delta \mu_{\alpha} = +15.3^{\circ} \pm 22.7^{\circ}$$

$$\Delta \mu_{\delta} = -0.44.3^{\circ} \pm 0.7.5^{\circ}$$

$$\Delta \alpha = -0''.526 \pm 0''.365$$

$$\Delta \delta = +0.0003^{\circ} \pm 0.0011^{\circ}$$

$$\Delta \mu = 0$$

$$= +0^{\circ} 5'.5 \pm 0^{\circ} 1'.1$$

$$= -6.14.0^{\circ} \pm 0.22.3^{\circ}$$

$$= -0.15.3^{\circ} \pm 0.22.7^{\circ}$$

$$= -0.46.0^{\circ} \pm 0.2.4^{\circ}$$

$$= -0''.736 \pm 0''.117$$

$$= -0.0010^{\circ} \pm 0.0004^{\circ}$$

$$= +0^{\circ}.04047 \pm 0^{\circ}.00184$$

$$[\mu_{\alpha}] = 7.88$$

1884-5-

$$\begin{aligned}
 d\delta &= -0^\circ 51'.0 \pm 0^\circ 2'.7 & = -0^\circ 50'.9 \pm 0^\circ 2'.5 \\
 d^{\circ}P &= -2 \ 24.7 \pm 0 \ 18.3 & = -2 \ 12.4 \pm 0 \ 16.9 \\
 dN &= -0 \ 9.7 \pm 0 \ 22.0 & = -0 \ 1.5 \pm 0 \ 20.1 \\
 df &= -0 \ 45.3 \pm 0 \ 2.6 & = -0 \ 44.1 \pm 0 \ 2.4 \\
 d\Delta &= +0''.749 \pm 0''.111 & = +0''.671 \pm 0''.103 \\
 de &= +0.0110 \pm 0.0003 & = +0.0111 \pm 0.0003 \\
 d\mu &= 0 & = -0''.00710 \pm 0''.00182
 \end{aligned}$$

1884-5-

In considering these two sets of corrections, we see that in all the oppositions, except 1879, the probable errors of the second set are the smaller, therefore it has been decided to use this latter set, giving us the following corrected elements.

§ 7. *Table C*

Epoch Oct. 0.0, 1875
 ϵ $63^{\circ} 7'.2 \pm 0^{\circ} 3'.2$
 ρ $72.504 \pm 0.20.2$
 A $122 \ 14.2 \pm 0 \ 24.7$
 β $5 \ 55.3 \pm 0 \ 3.7$
 Δ $215''.451 \pm 0''.135$
 μ $16^{\circ}.85024 \pm 0^{\circ}.00221$
 v 0.0015 ± 0.0008

Epoch Nov. 0.0, 1877
 ϵ $346^{\circ} 24'.8 \pm 0^{\circ} 6'.2$
 ρ $13 \ 17.7 \pm 0 \ 26.1$
 A $123 \ 15.5 \pm 0 \ 35.7$
 β $5 \ 54.5 \pm 0 \ 3.3$
 Δ $214''.244 \pm 0''.210$
 μ $16^{\circ}.86284 \pm 0^{\circ}.00289$
 v 0.0015 ± 0.0008

Epoch Jan. 0.0, 1878
 ϵ $94^{\circ} 29'.5 \pm 0^{\circ} 3'.2$
 ρ $86 \ 31.0 \pm 1 \ 1.7$
 A $122 \ 9.7 \pm 0 \ 15.0$
 β $5 \ 47.2 \pm 0 \ 2.6$
 Δ $214''.336 \pm 0''.137$
 μ $16^{\circ}.92637 \pm 0^{\circ}.00293$
 v 0.00789 ± 0.0004

Epoch Feb. 0.0, 1878
 ϵ $184^{\circ} 5'.8 \pm 0^{\circ} 2'.4$
 ρ $59 \ 18.3 \pm 0 \ 30.8$
 A $121 \ 20.6 \pm 0 \ 22.7$
 β $5 \ 53.0 \pm 0 \ 2.5$
 Δ $215''.331 \pm 0''.161$
 μ $16^{\circ}.86994 \pm 0^{\circ}.00279$
 v 0.00837 ± 0.0008

Epoch Jan. 0.0, 1881

δ $302^{\circ} 52.4 \pm 0^{\circ} 2.2$
 P $351 \quad 20.5 \pm 0 \quad 22.3$
 A $123 \quad 56.5 \pm 0 \quad 22.7$
 Δ 213.001 ± 0.001
 μ $16^{\circ} 96035 \pm 0^{\circ}.00184$
 e 0.0990 ± 0.0004

Epoch Jan. 0.0, 1885

δ $359^{\circ} 39.1 \pm 0^{\circ} 2.5$
 P $336 \quad 31.8 \pm 0 \quad 16.7$
 A $124 \quad 12.6 \pm 0 \quad 20.1$
 Δ 213.001 ± 0.001
 μ $16^{\circ} 91278 \pm 0^{\circ}.00182$
 e 0.1111 ± 0.0003

The set of elements, and to determine the most probable set of elements, the most probable set for the year, 1885.

By the method of least squares.

In attempting to determine the most probable set of elements, the method of least squares is used.

$$P = 82.00 - 21.00t$$

where t is the number of years from 1880.0.

The six values of P , give six equations of the form

$$x - y t + z t^2 = \Delta^{(2)},$$

from whose solution we obtain the values of x, y, z , which are substituted in the equation of P , give

$$P = \cos \theta = \cos \alpha - (\sin \alpha \cos \delta \cos \epsilon) + (\sin \alpha \cos \delta \sin \epsilon \sin \theta)$$

An attempt was made to express the residuals in the form

$$\Delta P = \epsilon \sin \Pi$$

where Π is the mean elongation of the heretofore mentioned planet & ϵ is a constant. The reason for this assumption is given in Professor Newcomb's paper referred to above.

The constant ϵ was well determined for the years that the mean T. lies within the years under consideration lies between the limits

90° and 270° , its sine comes out to be ± 1 . In the case of 90° and 270° , the sine is ± 1 , and the introduction of a great uncertainty into these respective values.

If Π is greater than 270° or the near immediate vicinity thereof, the calculation of Π is not possible, and the value of Π is not determined. The value of Π is not determined, and the value of Π is not determined. The results of the several steps in this attempt follow.

The assumed value of P is

$$P = 82.0 - 19.0t + 10^\circ \sin \Pi$$

where $\Pi = 187.9 + 19.5t$

The value of P is assumed to be

$$P = 82.0 - 19.0t + 10.0 \sin \Pi,$$

which gives $\Pi_1 = 188.33 + 16.665t$

$$I_2 = 82^{\circ}.50 - 15^{\circ}.137t + 30^{\circ}.14 \sin \Pi_2$$

$$\Pi_2 = 137^{\circ}.38 + 15^{\circ}.657t$$

A third approximation

$$I_3 = 83^{\circ}.35 - 13^{\circ}.766t + 38^{\circ}.26 \sin \Pi_3$$

$$\Pi_3 = 136^{\circ}.53 + 14^{\circ}.238t$$

The following table gives the residuals in each case, as also those in the case of I_0 on page 50.

The weights are obtained from the probable errors

I_0	I_1	I_2	I_3	I_4	I_5	I_6
1870	1870	1870	1870	1870	1870	1870
1871	1871	1871	1871	1871	1871	1871
1872	1872	1872	1872	1872	1872	1872
1873	1873	1873	1873	1873	1873	1873
1874	1874	1874	1874	1874	1874	1874
1875	1875	1875	1875	1875	1875	1875
1876	1876	1876	1876	1876	1876	1876
1877	1877	1877	1877	1877	1877	1877
1878	1878	1878	1878	1878	1878	1878
1879	1879	1879	1879	1879	1879	1879
1880	1880	1880	1880	1880	1880	1880

Date	$1 - \frac{r}{a}$	$\frac{r}{a} \cos \theta$	$\frac{r}{a} \sin \theta$	$\frac{r}{a} \sin^2 \theta$
1875.75	-0.01	+0.00	+0.00	+0.00
1876.00	+0.01	+0.00	+0.00	+0.00
1879.03	+0.01	+0.01	+0.00	+0.00
1880.03	+0.01	+0.00	+0.00	+0.00
1882.00	+0.01	+0.01	+0.00	+0.00
1883.00	+0.01	+0.00	+0.00	+0.00

Eccentricity.

In combining the different values of the eccentricity we obtain

$$e_0 = 0.09868 \pm 0.00002 - (0.01905 \pm 0.00003) \cos \pi_0$$

In the following table the weights are obtained in the manner described above.

λ	μ	α	β	γ
1875.75	2	0.1013	0.0762	+0.0051
1877.83	1	0.0905	0.0827	+0.0078
1879.83	5	0.0787	0.0790	-0.0001
1880.83	2	0.0837	0.0816	+0.0021
		0.0990	0.1025	-0.0025
	6	0.0801	0.0801	+0.0000

Mean longitude

If we neglect the periodic term, the most probable value of the mean longitude is found to be

$$\bar{O}_0 = 55^\circ.55 + 16^\circ.91868t$$

where t is the number of days from Jan. 0.0 1880.

Introducing the periodic term $e \sin \Pi_0$, we obtain

$$\bar{O}_1 = 55^\circ.19 + 16^\circ.90661t - 21^\circ.84 \sin \Pi_0$$

The remarks made with reference to the periodic term in the longitude of the peri-satellite apply equally well here.

The following table gives the values of E_0 and E_1 ,
 as a function of the mean longitude λ .

Lat.	λ	E (alt.)	E_0	E_1	$E - E_0$	$E - E_1$
Dec. 20, 1873	0	211.2	211.2	17.53	193.67	176.14
Nov. 30, 1877	5	204.20	204.7	201.77	2.43	2.43
Nov. 30, 1879	10	198.1	198.51	186.48	11.63	11.63
Nov. 30, 1880	15	192.0	175.75	182.77	16.25	9.23
Jan. 30, 1884	20	185.8	293.74	297.20	-13.40	-13.40
Jan. 30, 1885	25	179.5	5.98	1.35	-4.63	-4.63

From these results we see that while we can obtain expressions that will fairly well represent all the mean longitudes, and the longitudes of peri-saturnium, the observations here discussed do not enable us to distinguish sharply the uniform motion in each case from the periodic term, but if the observations be continued

... more opposition, in all parts
... will result.

The ...
...
...
...
variations of the elements.

The value found in this paper being

$$d_c = +0.01985 \cos \pi$$

$$d = +0.017 \cos \pi$$

is the value found by Professor Newcomb.

William L. C.

City of Baltimore September 12, 1855, and is the only son of Robert L. and Martha Chickering.

Early in life he entered the primary department of the City College in the summer of 1853.

He was a member of one of the Latin classes. In the fall of the same year, having received one of the scholarships intended for naturalists, he entered Johns Hopkins University. In 1856 he was graduated, receiving the degree of A. B., and was one of two to be appointed to a graduate scholarship.

In the fall of this year he entered for a post-graduate course, and the following year was made a Fellow in Mathematics.

In 1857 the Astronomical Board then





